IMPOSING ESSENTIAL BOUNDARY CONDITIONS IN MESH-FREE METHODS

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In spite of the important effort dedicated to mesh-free methods in the last decade (see [1,2] for a general presentation), there are still many aspects that require further research. For example, the imposition of essential boundary conditions in Galerkin based mesh-free methods is still an open topic. In mesh-free interpolation techniques, shape functions usually do not verify the Delta property. That is, the set of mesh-free shape functions is a partition of the unity, but the shape function associated to a particle does not vanish at other particles. Therefore, the imposition of Dirichlet boundary conditions is not trivial. Many specific techniques have been developed in the recent years in order to impose essential boundary conditions in mesh-free methods. Some possibilities are: (1) use Lagrange multipliers (Belytschko et al. 1994), (2) penalty methods (Zhu and Atluri 1998), (3) coupling with finite elements (Belytschko et al. 1995, Huerta and Fernández-Méndez 2000, Wagner and Liu 2001), (4) specially modified shape functions (Gosz and Liu 1996, Wagner and Liu 2000) or (5) use the Nitsche method (Griebel and Schweitzer 2002), among others. The aim of this work is to review and compare the most competitive techniques for the imposition of essential boundary conditions.

The method of Lagrange multipliers is one of the most widely used in the imposition of essential boundary conditions, because of its easy implementation and application to all kind of problems. Several possibilities can be considered for the interpolation of the Lagrange multiplier at the essential boundary. The point collocation technique is the easiest one and therefore it is the most popular. However, in any case, attention must be paid in the choice of the interpolation space for the Lagrange multiplier. The discretization of the multiplier must be accurate enough in order to obtain an acceptable solution, but the resulting system of equations turns out to be singular if the number of Lagrange multipliers is too large. In fact, the interpolation spaces for the Lagrange multiplier and for the principal unknown must verify an inf-sup condition, similar to the LBB condition, in order to ensure the convergence of the approximation. Another method based in a modification of the weak form is the Nitsche method. This modification depends only on the choice of one scalar parameter. The convergence of the method is ensured if this parameter is large enough. However, the method is not as general as the method of Lagrange multipliers, in the sense that the modification of the weak form is different, and must be deduced, for each particular problem. On the other hand, the possibility of coupling the mesh-free interpolation with finite elements in a neighborhood of the essential boundary, allows the application of all the techniques developed for the imposition of essential boundary conditions in the framework of finite elements. This implies the modification of the mesh-free code in order to include finite elements, but the modifications are made only at the interpolation level, and it can be easily applicable to all kind of problems.

References

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[2] W.K. Liu, T. Belytschko and J.T. Oden eds, "Meshless Methods", *Comp. Meths. Appl. Mech. Engr.*, v. 139, 1996.